

## EJERCICIOS DE LOGARITMOS Y EXPONENTES

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### PARTE I

En cada uno de los ejercicios, escribir la igualdad dada, en forma logarítmica

- $4^3 = 64 \equiv \log_4 64 = 3$
- $9^{\frac{1}{2}} = 3 \equiv \log_9 3 = \frac{1}{2}$
- $8^{\frac{2}{3}} = 4 \equiv \log_8 4 = \frac{2}{3}$
- $4^0 = 1 \equiv \log_4 1 = 0$
- $6^4 = 1296 \equiv \log_6 1296 = 4$
- $256^{\frac{1}{4}} = 4 \equiv \log_{256} 4 = \frac{1}{4}$
- $49^{\frac{1}{2}} = 7 \equiv \log_{49} 7 = \frac{1}{2}$
- $25^{-\frac{1}{2}} = \frac{1}{5} \equiv \log_{25} \frac{1}{5} = -\frac{1}{2}$
- $8^{-\frac{1}{3}} = \frac{1}{2} \equiv \log_8 \frac{1}{2} = -\frac{1}{3}$
- $13^4 = 28561 \equiv \log_{13} 28561 = 4$
- $5^{\frac{1}{2}} = \sqrt{5} \equiv \log_5 \sqrt{5} = \frac{1}{2}$
- $16^{\frac{3}{2}} = 64 \equiv \log_{16} 64 = \frac{3}{2}$
- $11^3 = 1331 \equiv \log_{11} 1331 = 3$
- $7^4 = 2401 \equiv \log_7 2401 = 4$
- $4^{\frac{5}{2}} = 32 \equiv \log_4 32 = \frac{5}{2}$
- $36^{\frac{1}{2}} = 6 \equiv \log_{36} 6 = \frac{1}{2}$
- $125^{-\frac{2}{3}} = \frac{1}{25} \equiv \log_{125} \frac{1}{25} = -\frac{2}{3}$
- $10^3 = 1000 \equiv \log_{10} 1000 = 3$
- $10^{-3} = 0.001 \equiv \log_{10} 0.001 = -3$
- $256 = 4^4 \equiv \log_4 256 = 4$
- $3^1 = 3 \equiv \log_3 3 = 1$
- $2^7 = 128 \equiv \log_2 128 = 7$
- $10^4 = 10000 \equiv \log_{10} 10000 = 4$
- $\left(\frac{1}{2}\right)^{-3} = 8 \equiv \log_{\frac{1}{2}} 8 = -3$
- $(\sqrt{3})^4 = 9 \equiv \log_{\sqrt{3}} 9 = 4$
- $x^1 = x \equiv \log_x x = 1$
- $x^0 = 1 \equiv \log_x 1 = 0$

28.  $x^y = m \equiv \log_x m = y$
29.  $n^m = p \equiv \log_n p = m$
30.  $t^p = q \equiv \log_t q = p$

## PARTE II

En cada uno de los ejercicios, escribir la igualdad dada en forma exponencial.

1.  $\log_2 256 = 8 \equiv 2^8 = 256$
2.  $\log_8 2 = \frac{1}{3} \equiv 8^{\frac{1}{3}} = 2$
3.  $\log_2 \frac{1}{8} = -3 \equiv 2^{-3} = \frac{1}{8}$
4.  $\log_3 27 = 3 \equiv 3^3 = 27$
5.  $\log_4 2 = \frac{1}{2} \equiv 4^{\frac{1}{2}} = 2$
6.  $\log_3 \frac{1}{81} = -4 \equiv 3^{-4} = \frac{1}{81}$
7.  $\log_2 \sqrt[4]{32} = \frac{5}{4} \equiv 2^{\frac{5}{4}} = \sqrt[4]{32} = \sqrt[4]{2^5}$
8.  $\log_{32} \frac{1}{4} = -\frac{2}{5} \equiv 32^{-\frac{2}{5}} = \frac{1}{4}$
9.  $\log_{64} 8 = \frac{1}{2} \equiv 64^{\frac{1}{2}} = \sqrt[2]{64} = 8$
10.  $\log_{81} 3 = \frac{1}{4} \equiv 81^{\frac{1}{4}} = (9^2)^{\frac{1}{4}} = 9^{\frac{2}{4}} = 9^{\frac{1}{2}} = \sqrt[2]{9} = 3$
11.  $\log_4 16 = 2 \equiv 4^2 = 16$
12.  $\log_8 64 = 2 \equiv 8^2 = 64$
13.  $\log_5 3125 = 5 \equiv 5^5 = 3125$
14.  $\log_3 \frac{1}{9} = -2 \equiv 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
15.  $\log_7 343 = 3 \equiv 7^3 = 343$
16.  $\log_4 \frac{1}{128} = -3.5 \equiv 4^{-3.5} = 4^{-\frac{7}{2}} = \frac{1}{4^{\frac{7}{2}}} = \frac{1}{4^3 \sqrt{4}} = \frac{1}{64(2)} = \frac{1}{128}$
17.  $\log_{10} 10000 = 4 \equiv 10^4 = 10000$
18.  $\log \frac{1}{100} = -2 \equiv 10^{-2} = \frac{1}{10^2} = \frac{1}{100}$
19.  $\log 0.0001 = -4 \equiv 10^{-4} = 0.0001$
20.  $\log 10 = 1 \equiv 10^1 = 10$
21.  $\log_e 1 = 0 \equiv e^0 = 1$
22.  $\log_3 \frac{1}{243} = -5 \equiv 3^{-5} = \frac{1}{3^5} = \frac{1}{243}$
23.  $\log_7 1 = 0 \equiv 7^0 = 1$
24.  $\log_{25} 1 = 0 \equiv 25^0 = 1$
25.  $\log_{11} 11 = 1 \equiv 11^1 = 11$
26.  $\log_4 \frac{1}{64} = -3 \equiv 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$
27.  $\log_e e = 1 \equiv e^1 = e$

28.  $\log_e w = r \equiv e^r = w$   
 29.  $\log_t x = n \equiv t^n = x$   
 30.  $\log_k m = f \equiv k^f = m$

PARTE III

Calcule las expresiones dadas.

1.  $\log 100 = 2$
2.  $\log 0.25 = \log \frac{1}{4} = \log 1 - \log 4 = 0 - \log 4 = -\log 4$
3.  $\log \sqrt{30} = \log(30)^{\frac{1}{2}} = \frac{1}{2}(\log 30) = \frac{1}{2}(\log(3 \cdot 10)) = \frac{\log 3}{2} + \frac{\log 10}{2} = \frac{\log 3}{2} + \frac{1}{2}$
4.  $\log_4 \frac{1}{16} = \log_4 1 - \log_4 16 = 0 - 2 = -2$
5.  $\log_2 32 = 5$
6.  $\log_8 64 = 2$
7.  $\log 0.0001 = -4$
8.  $\log_3 81 = 4$
9.  $\log_2 12.5 = \log_2 \frac{25}{2} = \log_2 25 - \log_2 2 = \log_2 5^2 - 1 = 2 \log_2 5 - 1$
10.  $\log_{\sqrt{5}} 625 = 8$
11.  $\log_{\frac{1}{3}} 102 = \log_{\frac{1}{3}} 3 \cdot 34 = \log_{\frac{1}{3}} 3 + \log_{\frac{1}{3}} 34$   
 11.  $\log_{\frac{1}{3}} 102 = \frac{\log 102}{\log \frac{1}{3}}$
12.  $\log_{\sqrt[3]{10}} 0.0159 = \frac{\log 0.0159}{\log \sqrt[3]{10}}$
13.  $\log_{0.48} 40.32 = \frac{\log 40.32}{\log 0.48}$
14.  $\log_{\sqrt{2}} \sqrt{3} = \frac{\log \sqrt{3}}{\log \sqrt{2}}$
15.  $\log_{1.05} 50 = \frac{\log 50}{\log 1.05}$
16.  $\log_{\sqrt{7}} \sqrt{49} = 2$
17.  $\log_{\frac{5}{9}} \frac{12}{13} = \frac{\log \frac{12}{13}}{\log \frac{5}{9}}$
18.  $\log_2 \sqrt[4]{\frac{1}{8}} = \log_2 \frac{1}{2^{\frac{3}{4}}} = \log_2 1 - \log_2 2^{\frac{3}{4}} = 0 - \frac{3}{4} = -\frac{3}{4}$
19.  $\log_3 \sqrt[5]{\frac{1}{243}} = \log_3 \left(\frac{1}{3^5}\right)^{\frac{1}{5}} = \log_3 \frac{1}{3} = \log_3 1 - \log_3 3 = 0 - 1 = -1$
20.  $\log \frac{\sqrt[4]{1000}}{(0.001)^2} = \log \frac{10^{\frac{3}{4}}}{\left(\frac{1}{10^3}\right)^2} = \log \frac{10^{\frac{3}{4}}}{\frac{1}{10^6}} = \log \frac{(10^{\frac{3}{4}})(10^6)}{1} = \log 10^{\frac{27}{4}} = \log 10^{\frac{27}{4}} = \frac{27}{4}$
21.  $\log_2 \frac{\sqrt{512} \left(\frac{1}{4}\right)^5}{64 \cdot \left(\frac{1}{8}\right)^2} = \log_2 \frac{2^{\frac{9}{2}} \left(\frac{1}{2^2}\right)^5}{64 \cdot \frac{1}{64}} = \log_2 \frac{2^{\frac{9}{2}}}{2^{10}} = \log_2 2^{-\frac{11}{2}} = -\frac{11}{2}$

#### PARTE IV

En cada uno de los ejercicios, determinar el valor de  $a$ ,  $x$  y  $y$  ( $a > 0$ ).

1.  $\log_9 81 = x \Rightarrow x = 2$
2.  $\log_a 4 = \frac{1}{2} \Rightarrow a = 16$
3.  $\log_{64} 2 = y \Rightarrow y = \frac{1}{6}$
4.  $\log_4 \frac{1}{16} = x \Rightarrow x = -2$
5.  $\log_3 x = -3 \Rightarrow x = \frac{1}{27}$
6.  $\log_a 121 = 2 \Rightarrow a = 11$
7.  $\log_5 0.04 = y \Rightarrow -2$
8.  $\log_{\frac{1}{5}} 125 = x \Rightarrow x = -3$
9.  $\log_a \frac{1}{9} = -2 \Rightarrow 3$
10.  $\log_{0.5} x = -2 \Rightarrow x = 4$
11.  $\log_8 x = 4 \Rightarrow x = 4096$
12.  $\log_{10} x = -3 \Rightarrow x = 0.001$
13.  $\log_{0.25} 256 = y \Rightarrow -4$
14.  $\log y = -5 \Rightarrow 0.00001$
15.  $\log x = -2 \Rightarrow 0.01$
16.  $\log_e x = 2 \Rightarrow x = e^2$
17.  $\log_e 1 = x \Rightarrow x = 0$
18.  $\log_e y = 1 \Rightarrow y = e$
19.  $\log_a 14641 = 4 \Rightarrow a = 11$
20.  $\lim_a \frac{1}{625} = 4 \Rightarrow a = \frac{1}{5}$
21.  $\lim_a \frac{1}{1000} = -3 \Rightarrow a = 10$

#### PARTE V

Expresé el logaritmo dado en términos de los logaritmos de  $x, y$  y  $z$

$$1. \log_a \frac{xy}{z} = \log_a x y - \log_a z =$$

$$\log_a x + \log_a y - \log_a z$$

$$2. \log_a \frac{z\sqrt{y}}{\sqrt{x}} = \log_a z\sqrt{y} - \log_a \sqrt{x} =$$

$$\log_a z + \log_a \sqrt{y} - \log_a \sqrt{x} = \log_a z +$$

$$\log_a y^{\frac{1}{2}} - \log_a x^{\frac{1}{2}} = \log_a z + \frac{1}{2} \log_a y - \frac{1}{2} \log_a x$$

$$3. \log_a \frac{xy^2}{z^5} = \log_a xy^2 - \log_a z^5 = \log ax + 2 \log ay - 5 \log az$$

$$4. \log_a \frac{\sqrt[3]{xy^4}}{\sqrt[3]{z}} = \log_a \sqrt[3]{xy^4} - \log_a \sqrt[3]{z} = \log a 3x + \log ay 4 - \log a 3z = 1 3 \log ax + 4 \log ay - 1 3 \log az$$

$$5. \log_a \frac{x^2y}{z^3} = \log_a x^2y - \log_a z^3 = \log_a x^2 + \log_a y - \log_a z^3 = 2 \log_a x + \log ay - 3 \log az$$

$$6. \log_a \frac{x^3y^2}{z^5} = \log_a x^3y^2 - \log_a z^5 = \log_a x^3 + \log_a y^2 - \log_a z^5 = 3 \log_a x + 2 \log ay - 5 \log az$$

$$7. \log_a \frac{\sqrt{xz^2}}{y^4} = \log_a \sqrt{xz^2} - \log_a y^4 =$$

$$\log_a x + \log_a z^2 - \log_a y^4 = 1\log_a x + 2\log_a z - 4\log_a y$$

$$8. \log_a x \cdot \sqrt[3]{\frac{y^2}{z^4}} = \log_a x + \log_a \sqrt[3]{\frac{y^2}{z^4}} =$$

$$\log_a x + \frac{1}{3} \log_a \frac{y^2}{z^4} = \log_a x + \frac{1}{3} \log_a y^2 -$$

$$1\log_a z^4 = \log_a x + 2\log_a y - 4\log_a z$$

$$9. \log_a \sqrt[3]{\frac{x^2}{yz^5}} = \frac{1}{3} \log_a \frac{x^2}{yz^5} = \frac{1}{3} \log_a x^2 -$$

$$1\log_a yz^5 = 2\log_a x - 1\log_a y - 1\log_a z^5$$

$$= 2\log_a x - 1\log_a y - 5\log_a z$$

$$10. \log_a \frac{\sqrt{xy^6}}{\sqrt[3]{z^2}} = \log_a \sqrt{xy^6} - \log_a \sqrt[3]{z^2} =$$

$$1\log_a x + 1\log_a y^6 - 2\log_a z = 1\log_a x + 6\log_a y - 2\log_a z$$

$$= 1\log_a x + 6\log_a y - 2\log_a z$$

$$11. \log_a \sqrt{x\sqrt{yz^3}} =$$

$$\begin{aligned} \frac{1}{2} \log_a x (yz^3)^{\frac{1}{2}} &= \frac{1}{2} \log_a x + \frac{1}{2} \log_a (yz^3)^{\frac{1}{2}} = \\ 12 \log_a x + 14 \log_a yz^3 &= 12 \log_a x + 14 \log_a y + \\ 14 \log_a z^3 &= 12 \log_a x + 14 \log_a y + 34 \log_a z \end{aligned}$$

$$12. \log_a \sqrt[3]{x^2 y \sqrt{z}} = \frac{1}{3} \log_a x^2 y \sqrt{z} =$$

$$\begin{aligned} 13 \log_a x^2 + 13 \log_a y + 13 \log_a z &= 23 \log_a x + 1 \\ 3 \log_a y + 16 \log_a z \end{aligned}$$

13.